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Decay of cosmological constant in self-consistent inflation

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Abstract. The symmetric vacuum state in gauge theories with spontaneous symmetry breaking is symmetric in both internal and space-time variables. We consider this vacuum state as a Bose condensate of physical Higgs particles, defined over an asymmetric vacuum state, and identify the energy density of their self-interaction with the cosmological constant Λ in the Einstein equation. In this picture, spontaneous symmetry breaking proceeds as Λ decay. Decoherence of coherent oscillations of a scalar field in the course of Λ decay provides the effective mechanism for damping of coherent oscillations, leading to the regime of slow evaporation of a Bose condensate. This mechanism is responsible for self-consistent inflation without fine-tuning of the potential parameters. The physical self-consistency in this model is provided by incorporating the origin of the cosmological constant in the dynamics of spontaneous breaking of particle symmetries.

1 Introduction

Developments in particles and quantum field theory, as well as the confrontation of models with the observations in cosmology, compellingly indicate that the cosmological constant Λ ought to be treated as a dynamical quantity (for a recent review, see [31]). Anything which contributes to the energy density $\rho_{\rm vac}$ in such a way that $T^{\mu\nu}_{\rm vac} = \rho_{\rm vac} g_{\mu\nu}$ behaves like a cosmological term in the Einstein equations. At the very early stage of the evolution of the universe, the cosmological term had to be very large in order to drive inflation, thus providing the reason for the expansion of the universe and explaining its isotropy and large scale homogeneity [19–21, 11, 26, 29]. Now the cosmological term is estimated and compared with the mean density in the universe [1, 25, 30].

Several mechanisms have been proposed involving and supporting a negative vacuum energy density growing with time to cancel the initial pre-existing positive cosmological term and to drive the ultimate value of Λ toward zero. All those mechanisms utilize the basic property of the de Sitter space-time – its quantum and semiclassical instabilities.

Starobinsky has investigated the instability of the de Sitter space in the theory, including the higher-derivative terms in the Einstein equations which arise from one-loop quantum contributions of massless conformal matter fields [37].

Myhrvold has discovered the quantum instability of the de Sitter space-time, which leads to spontaneous decay of the effective value of Λ due to the fact that the stress-energy tensor of a quantum field in de Sitter space takes the form of a cosmological term growing with time with a sign opposite to that of the background space-time. Myhrvold has examined the particle production mechanism for scalar $\lambda\phi^4$ theory in de Sitter space and found that the rate of particle production is proportional to the number of particles present, yielding an exponentially increasing rate of production – runaway particle production – due to the quantum instability of the de Sitter space-time [28].

Ford has investigated in a most rigorous way the case of the massless minimally coupled scalar field and has shown that the vacuum expectation value of its stress-energy tensor can contain a term which is proportional to the metric tensor $g_{\mu\nu}$, with a coefficient growing linearly with time and acting in the direction of canceling the pre-existing cosmological constant [17].

Dolgov has considered the model in which a classically unstable scalar field, non-minimally coupled to gravity, develops a negative energy density canceling the initial positive value of the cosmological constant [9]. In this approach, the Newton gravitational constant G becomes the effective coupling $G_{\rm eff}$ which is quenched by the proposed mechanism: $G_{\rm eff} \to 0$ as $t \to \infty$, while $D_{\rm eff}/G_{\rm eff} \sim -10^{-10}\,{\rm yr}^{-1}$ which strongly contradicts the upper limits from Viking radar ranking and from lunar laser ranging [34]. An extension of this mechanism to higher spin fields removes this problem [10], but retains the other difficulty of this approach: The non-cancelled part of Λ remains permanently of the order of the Ricci tensor while it needs

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to be much less during the matter dominated epoch to achieve sufficient growth of the perturbations [34].

Mottola has considered the decay of the invariant vacuum state as appropriate for inflationary models (corresponding to the Unruh vacuum for the case of the Schwarzschild geometry) via emission of scalar particles due to the Hawking effect in de Sitter space. In this model, the created pairs have a stress-energy tensor which supports a first order decrease of the effective cosmological constant independently of any matter phase transition [27].

Hiscock has analyzed the compatibility of models dominated by a decaying cosmological term Λ with realistic cosmological models satisfying observational constraints [22]. Gravity-mediated decay mechanisms [28, 17, 27] require that Λ decays slowly and that Λ always dominates over the matter and radiation, so that the universe is approximately described by the de Sitter metric. Confronting the ultimate parameters of cosmological models with Λ decaying with nucleosynthesis constraints, Hiscock concluded that it is unlikely that a realistic cosmology can be constructed which allows Λ to decay from an initially pre-existing large value and at the same time leads to the universe being "even remotely like our own" [22].

On the other hand, it is possible to incorporate the origin of an initial Λ into the general context of scalar field dynamics. In our papers [12, 6] we have shown that the process of the emergence of massive scalar particles in (from) the de Sitter vacuum looks like the evaporation of a Bose condensate. In our paper [13] we proposed to treat the classical scalar field as the Bose–Einstein condensate of the physical quanta of this field defined over its ground state (true vacuum).

In scalar field dynamics the potential of the scalar field $V(\phi)$ plays the role of an effective cosmological constant in regimes where its derivatives are close to zero. Starting from such a regime as the initial state, the field equations typically lead to successive coherent field oscillations [24, 29]. In the context of inflationary models with the effective Λ related to the inflaton field in the slow-rolling regime, the further decay of coherent oscillations involves inflaton interactions with other fields in preheating models [23, 36, 8, 5, 39].

In our paper [13] we proposed the model of self-consistent inflation in which the same self-interacting scalar field is responsible for both the initial value of Λ and for its further decay. The key point in our approach is the involvement of both space-time and particle internal symmetries. In gauge theories of elementary particles, the mechanisms of spontaneous breaking of unification symmetry imply that the unbroken symmetry state is a false vacuum state. This vacuum is at the same time the highly symmetric state of space-time geometry invariant under the de Sitter group. This vacuum state provides the proper choice for the intial conditions maximally symmetric in both space-time and internal particle variables. In [13] we identified this state with the Bose–Einstein condensate of physical Higgs particles whose self-interaction energy density corresponds to a scalar field potential $V(0)$ in the state with unbroken symmetry. The process of decay of the inflationary vacuum was then considered as the evaporation of a Bose condensate [12], which results in the unambigous choice of the asymmetric vacuum state, as well as in a non-slow-rolling transition to the standard FRW cosmological model, which in turn allows us to avoid the fine-tuning in the initial conditions for inflation. The dynamics of the cosmological term is directly related to the hierarchy of particle symmetries breakings, which makes such an approach physically self-consistent [13].

In our paper [14] we identified the cosmological constant Λ with the energy density of the self-interaction of scalar bosons bound in a condensate. In this approach, Λ decay provides a dynamical realization of spontaneous symmetry breaking. We also found the mechanism for the decoherence of coherent oscillations of a scalar field in the course of Λ decay.

In the present paper we investigate in detail the kinetics of Λ decay and the back reaction of decay products on the Λ dynamics. We show that effective kinetic damping of coherent scalar field oscillations results in the mechanism of slow evaporation, which seems to be generic for the dynamics of a Bose condensate. The slow evaporation regime is found here for a wide range of possible parameters of particle interactions. This mechanism provides the inflationary regime without fine-tuning of the potential parameters. Preliminary estimates show that this model is able to incorporate reheating, the proper density fluctuation spectrum and the non-zero value of the cosmological constant today.

Our paper is organized as follows. In Sect. 2 we address the question of the origin of Λ for the case of applying the Higgs mechanism in the inflationary cosmology. In Sect. 3 we consider the decay of a Bose condensate defined over the asymmetric (true) vacuum. In Sect. 4 we calculate Λ decay as the conversion of the vacuum energy in the condensate into the free massive Higgs particles which later decays into lighter species. Section 5 contains a summary and discussion.

2 The origin of Λ in Higgs field dynamics

In the books on inflationary cosmology we can read that, if a scalar field were displaced from the minimum of its effective potential, an enormous vacuum energy would exist, of order 10^{15} GeV for the GUT scale. It is this energy that powers inflation. Nevertheless, while inflation offers a solution to all well-known cosmological puzzles, it sheds no light on the problem of the cosmological constant itself [24].

On the one hand, in the most typical inflationary scenario a scalar field ϕ, after a slow-rolling period, approaches the minimum of its potential, which "steepens" near the minimum, and begins to oscillate around $\phi = \phi_0$ with a time scale short as compared to the expansion time and determined by the curvature of the potential. The enormous vacuum energy of the ϕ field then exists in the form of spatially coherent oscillations, corresponding to a condensate of zero-momentum ϕ particles.

Particle creation, or equivalently the decay of ϕ particles into the other species, to which they couple, will damp these oscillations, and as the decay products thermalize, the universe is reheated [24].

On the other hand, as a condensate of the Higgs field ($\phi = \phi_0$) develops after sufficient expansion and cooling the universe, the initial vacuum energy density (Λ), which should be enormous, has to coincide with that of the latter developed condensate with an enormous accuracy. The conversion of the vacuum energy stored in coherent oscillations of the field ϕ looks straightforward. The problem of Λ is related to the previous stage – the accelerated expansion driven by $V(0) = \Lambda$. During the whole period of slow rolling the vacuum energy density is $V(\phi)$, and $V(\phi)$ is almost constant, $V(\phi) \simeq V(0)$, which corresponds to zero field ϕ. Then it must be converted, at an extremely short time scale, into the energy density of the condensate of the ϕ particles, with extraordinary precision. One can hardly believe in such a superfine-tuning, and attempts have been made (outlined briefly in Sect. 1) to find some fundamental mechanism for the cancelation of Λ. Let us try to look for a natural way out of this puzzle, related to the stage of inflationary expansion. Let us carefully analyze the Higgs mechanism as applied to the inflationary picture.

In theories with spontaneous symmetry breaking, the vacuum expectation value of the Higgs field, which couples to bosons and fermions to give them masses, plays the role of an order parameter. The occurrence of a nonzero vacuum expectation value in the asymmetric vacuum state, $\langle\phi\rangle = \phi_0$, is interpreted as the development of a condensate of ϕ particles. That the vacuum is filled with a condensate of Higgs ϕ particles leads to spontaneous symmetry breaking [33,24]. It seems that the origin of the problem is hidden in this picture. ϕ particles are not physical particles, they are tachyonic with imaginary mass, which reflects the instability of the symmetric vacuum state of the theory. In the symmetric, physically stable, vacuum state (the state with the unbroken symmetry) the physical particles are χ particles, related to ϕ particles by $\phi = \phi_0 + \chi$. It is the χ particles that acquire a mass by the Higgs mechanism, and whose vacuum state is the true vacuum with zero potential and with zero expectation value, $\langle\chi\rangle = 0$. Therefore, in terms of the χ particles, the true vacuum of theories with spontaneous symmetry breaking cannot be a condensate, but rather we would have to treat the symmetric vacuum state of a theory as the condensate of χ particles. Replacing the ϕ condensate by a χ condensate sheds some light on the source of Λ, which in the ϕ condensate picture, where Λ is related to the state of zero field [21], looks mysterious. In the χ condensate picture, Λ is related to the non-zero value of the field χ as its energy density in the symmetric state for which further decay is naturally related to the dynamics of spontaneous symmetry breaking.

At the moment such a revision of the definition of a condensate may seem to be just a "solution" of the Λ problem, but let us try to proceed with this consequently. In the state with unbroken symmetry, the vacuum expectation value for the χ field is $\langle\chi\rangle = -\phi_0$, and then $V(-\phi_0)$ is directly the energy density of the χ condensate, which dynamically plays the role of the cosmological term which has to decay naturally when the field χ evolves towards the true vacuum $\langle\chi\rangle = 0$. The highly symmetric de Sitter state, corresponding to $V(\chi) = V(-\phi_0)$, is the vacuum state in both space-time and internal degrees of freedom. In the context of spontaneous symmetry breaking, such a choice provides a physically self-consistent way of identifying Λ as corresponding to the state with unbroken symmetry, but with non-zero vacuum expectation value of the scalar field χ. Considering $V(-\phi_0)$ as the energy density of the Bose condensate of the physical χ particles defined over the true vacuum state of the theory, $\langle\chi\rangle = 0$, we can treat the instability of the de Sitter vacuum state relative to particle creation as a natural realization of spontaneous symmetry breaking.

In terms of the χ field, the negative sign of the "mass term" $V''(\chi)$ in the state $\langle\chi\rangle = -\phi_0$ corresponds to χ particles bound by their self-interaction within a condensate, $\langle\chi\rangle = -\phi_0$. The development of instability proceeds as the evolution of the χ field from the initial bound state $\langle\chi\rangle = -\phi_0$ to the true vacuum state $\langle\chi\rangle = 0$. The mass of the physical χ particles is defined over this true vacuum state for free χ particles going out of the condensate. The process of the emergence of massive scalar particles in (from) the de Sitter vacuum looks like evaporation of a Bose condensate [12], and the presence or absence of a stage of slow rolling, as well as its duration, is determined by the dynamics of the scalar field in the de Sitter space. Such an approach reveals the deep connection between fundamental internal particle symmetries and the global structure of space-time.

Let us illustrate this picture by the simplest example of the Higgs field:

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \phi_0^2)^2. \tag{1}$$

In terms of the ϕ field it corresponds to the wrong sign of the mass term and thus to the unphysical character of the ϕ particles. In terms of the χ field the potential takes the form

$$V(\chi) = \lambda\phi_0^2\chi^2 + \lambda\phi_0\chi^3 + \frac{\lambda}{4}\chi^4. \tag{2}$$

In the state with unbroken symmetry $\langle\chi\rangle = -\phi_0$, $\langle\chi^2\rangle = \phi_0^2$, $\langle\chi^3\rangle = -\phi_0^3$, and the second derivative of the potential ("mass term") is $V''(\langle\chi\rangle = -\phi_0) = -\lambda\phi_0^2$, which corresponds to χ particles out of the mass surface (four-momentum $k_\mu \to 0$) bound in a condensate. The energy density of the condensate of the χ particles is given by

$$\langle V(\chi)\rangle = \frac{\lambda}{4}\phi_0^4. \tag{3}$$

This constant term, playing the role of the effective cosmological constant Λ in the Einstein equation, is identified in such an approach with the energy density of the condensate of the χ particles (3), being precisely the energy density of their self-interaction [14].

The Lagrangian of the Higgs field in this simple model is given by

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\chi_{;\mu}\chi_{;\nu} - \frac{\lambda}{4}(2\phi_0\chi + \chi^2)^2\right], \tag{4}$$

where we dropped, for simplicity, the indices of the internal variables.

The perturbations over the state with unbroken symmetry $\langle\chi\rangle = -\phi_0$ are described by the potential

$$V(\delta) = \frac{\lambda}{4}\phi_0^4 - \frac{\lambda}{2}\phi_0^2\delta^2 + \frac{\lambda}{4}\delta^4, \tag{5}$$

which correctly reflects the instability of the state with unbroken symmetry. The wrong sign of the mass term corresponds to the existence of the growing mode of the perturbation δ over the symmetric vacuum state, leading to its decay.

The perturbations determined over the asymmetric true vacuum state $\langle\chi\rangle = 0$, are described by the potential

$$V(\delta) = \lambda\phi_0^2\delta^2 + \lambda\phi_0\delta^3 + \frac{\lambda}{4}\delta^4. \tag{6}$$

It describes physical particles with masses of Higgs bosons of $m = (2\lambda)^{1/2}\phi_0$, self-interacting and interacting with the condensate. The term $\lambda\phi_0\delta^3$ corresponds to the decay of the condensate by the reaction $\phi \to 3\delta$, and describes evaporation of the Higgs bosons from the condensate of χ particles. The term $\lambda\phi^4/4$ reproduces the runaway particle production discovered by Myhrvold [28]. This differs from the Myhrvold result in the origin of Λ. The gravity-mediated decay of Λ in the Myhrvold approach is particle creation by a gravitational field generated by a pre-existing Λ which is not related to the created particles. In our approach, Λ is the energy density of the same particles bound within a condensate.

The difference of our approach from that of Mottola is that in Mottola's model particles are created by the gravitational field due to the Hawking mechanism which puts a constraint on the masses of the created particles $m_p \sim T_{\rm H} \sim H$ and leads to the exponential suppression of creation of particles with Higgs masses $m \gg H$ [27]. In fact, Mottola considered Hawking quantum evaporation of light scalar particles from the de Sitter horizon, while we are considering evaporation of Higgs bosons, with $m \gg H$, from the bound state inside a condensate into the free states.

Our approach differs also from the Parker and Zhang theory of a relativistic condensate as a source of inflation, in which the cosmological constant is set equal to zero for all times, the condensate is charged, and its decay proceeds via slow rolling [32]. In our model, the condensate of the χ particles is essentially globally neutral, since it corresponds to the state with unbroken symmetry – the totally symmetric state in both space-time and internal degrees of freedom. The condensate decays by evaporation (as well as by runaway production of χ particles which is exponentially suppressed [27,13]), and thus by the conversion of energy of an initial globally neutral state into the thermal energy of χ particles defined over their true vacuum. "Charge" has to appear when symmetry breaking completes and Higgs particles decay into the other species.

In the case of perfect efficiency of conversion of initial vacuum energy into radiation, the reheat temperature $T_{\rm RH} \sim m$, and the rate of decreasing of the vacuum energy is given by

$$\frac{\mathrm{d}\rho_{\rm vac}}{\mathrm{d}t} = -3H(\rho_{\rm r} + p_{\rm r}), \tag{7}$$

with

$$\rho_{\rm r} = \sigma T^4; \quad p_{\rm r} = \frac{1}{3}\sigma T^4, \tag{8}$$

where $\sigma = \pi^2 g/30$, and g counts the total number of effectively massless degrees of freedom. Then

$$\rho_{\rm vac} = \rho_0\left(1 - \frac{t}{\tau}\right)^2. \tag{9}$$

The characteristic time scale τ is given by

$$\tau = \frac{1}{16}\sqrt{\frac{3}{\pi}}\frac{\phi_0 m_{\rm Pl}^2}{\sigma m^3}\tau_{\rm Pl}. \tag{10}$$

Even in this simplest model with fastest decay, the e-folding number,

$$N_{\rm e} = \frac{1}{32\sigma\lambda}, \tag{11}$$

is sufficient for reasonable values of the self-interaction coupling λ. Let us emphasize here what the difference is between self-consistent inflation and inflation with slow rolling. In new inflation, the stage of slow rolling is in fact imposed on the model by choosing a potential with a very flat behavior near the symmetric state. The field then evolves down the hill from the symmetric state to the asymmetric one. In self-consistent inflation, evaporation of the condensate acts in such a way as to lower the potential near its symmetric state – not the field evolves along the potential of the imposed form, but the potential form itself evolves in the course of evaporation; the inflationary stage is not due to slow rolling, but due to a change of the form of the potential in the course of Λ decay by condensate evaporation. In Sect. 4 we show how the potential evolves as a result of the back reaction of the products of evaporation.

3 Decay of Bose condensate into free χ particles

Roughly, the scenario of Λ decay looks as follows. Within a χ condensate, χ particles have momenta $k \to 0$. Indeed, the Klein–Gordon equation for the potential (3) in the condensate regime ($\chi = -\phi_0; V'(\chi) = 0$) reduces to $\ddot{\chi} + 3H\dot{\chi} + \Gamma\dot{\chi} = 0$, where Γ is the decay rate. Its solution reads $\chi = -\phi_0 - \phi_0 \mathrm{e}^{-(3H+\Gamma)t}$. The time-dependent mode decays rapidly for any Γ stabilizing the particle state in the condensate with zero momentum ($\dot{\chi} \to 0$). This means that the χ condensate is the classical collective state within which the decay of the χ particles is impossible. (Their de Broglie wavelengths by far exceed the horizon, so that asymptotic states cannot be defined for the quantum transitions corresponding to decay inside a condensate.) Therefore, the decay of the χ particles occurs in free states which correspond to coherent field oscillations.

The fluctuations $\delta = \chi + \phi_0$ over the state with unbroken symmetry, $\langle\chi\rangle = -\phi_0$, are described by the potential (5). The evolution of the fluctuations is governed by the Klein–Gordon equation,

$$\ddot{\delta} + 3H\dot{\delta} - \frac{m^2}{2}\delta + \lambda\delta^3 = 0. \tag{12}$$

Let us estimate the characteristic time scale for the linear stage of development of the instability, neglecting the δ^3 term and taking into account that in our case ($\phi_0 \ll m_{\mathrm{Pl}}$)

$$\frac{m}{H} = \sqrt{\frac{3}{\pi}}\frac{m_{\mathrm{Pl}}}{\phi_0} \gg 1. \tag{13}$$

In this approximation, the solution to (12) is given by

$$\delta(t) = C_1 \mathrm{e}^{-(m/\sqrt{2})t} + C_2 \mathrm{e}^{(m/\sqrt{2})t}. \tag{14}$$

The growing mode of $\delta(t)$ corresponds to the decay of the condensate with a characteristic time scale of $\tau \sim m^{-1}$. To estimate the efficiency of the decay, we introduce the fluctuation density

$$n_\delta = \frac{m}{4}\delta^2. \tag{15}$$

Then

$$V(\delta) = \frac{\lambda}{4}\phi_0^4 - mn_\delta + \frac{4\lambda {n_\delta}^2}{m^2}. \tag{16}$$

The latter term can be interpreted as the self-interaction of the fluctuations with reaction rate $4\lambda/m^2$. In the minimum of the potential (16), the fluctuation density is $n_\delta = m^3/8\lambda$, which corresponds, by (15), to $\delta^2 = \phi_0^2$. We see that the stationary distribution of fluctuations around $\delta = \phi_0$ looks like a gas of χ particles with masses $m = (2\lambda)^{1/2}$ ϕ_0, evaporated from the "unphysical" state $k_\mu \to 0$ into the physical coherent ground state $k^2 = m^2$. The potential (16) achieves its minimum $V(\delta) = 0$, which corresponds to the total decay of the Λ condensate into a coherent state of physical χ particles, in a characteristic time $\tau \sim m^{-1} \ll H^{-1}$. So we must not worry about how to get efficient Λ decay, but rather about how to make this process last longer. In the next section we show qualitatively that this can in principle be achieved by the effective decoherence of the coherent state via back reaction of the decay products. The detailed quantum field analysis of the evolution of coherent states and of the mechanisms of their decoherence is currently under investigation [15].

4 Λ dynamics in slow evaporation regime

We see that identifying $V(-\phi_0)$ with the energy density of the self-interaction of the physical χ particles bound inside a Bose condensate, we can treat the instability of the de Sitter vacuum as a dynamical realization of symmetry breaking. The process of Λ decay proceeds in this picture as Bose condensate evaporation.

Higgs bosons are generally unstable. Both χ particles and products of their decay interact with Hawking radiation from the de Sitter horizon and with each other. This leads not only to decoherence of the χ particles but also to the appearance of relativistic particles over the condensate with the effective equation of state $p = \varepsilon/3$. The presence of a relativistic gas in thermodynamic equilibrium with the condensate facilitates its further decay.

The dynamical role of the not exponentially small thermal component in the evolution of an inflaton field has been studied by Berera and Li-Zhi Fang [3] and then was incorporated into the scenario called "warm inflation" [4]. The classical equation of motion for a scalar field φ in the de Sitter universe reads [24]

$$\ddot{\varphi} + 3H\dot{\varphi} + \Gamma_\varphi\dot{\varphi} + V'(\varphi) = 0.$$

The friction term Γ_φ is introduced phenomenologically to describe the decay of the inflaton field φ due to its interaction with the thermal component. Berera and Li-Zhi Fang have shown, first, that if $\Gamma_\varphi \sim H$, then the thermal component can play an essential role in the generation of primordial density fluctuations [3]. Second, in the regime $\Gamma_\varphi \gg H$, the thermal component becomes dominant in the equation of motion, leading to warm inflation [4]. The question of the self-consistency of this regime as governed by thermal components has been addressed in [4,38,2].

In our case, the mechanism leading to this condition, which we write as $\Gamma_\chi \gg H$, is not related to thermal components. The mechanism leading naturally to the regime $\Gamma_\chi \gg H$ is the effective friction due to the interaction of free particles with the ensemble of χ particles bound in the condensate with a large occupation number, which dampens oscillations and leads to the stationary regime of slow evaporation of the χ condensate responsible for the inflationary stage. Let us show that decoherence of the χ particle states and back reaction of the relativistic products of their decay lead to the effective damping of the fluctuations. The kinetic equation can be written in the standard way (see, e.g., [24]). The kinetic equation describing the growth and decay of the fluctuations reads

$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} = mn_\chi - \Gamma n_\chi - n_{\mathrm{r}}n_\chi\sigma - 3Hn_\chi, \tag{17}$$

where σ is the cross-section of interactions of the χ particles with relativistic products of their decay. In units $\hbar = c = 1$ the reaction rate in the kinetic equations coincides with σ. The first term on the right hand side describes the creation of χ particles, the second their decay, the third their interaction with products of decay, and the fourth their redshift.

The kinetic equation for the products of decay, effectively relativistic matter with the equation of state $p = \varepsilon/3$, reads

$$\frac{\mathrm{d}n_{\mathrm{r}}}{\mathrm{d}t} = -3Hn_{\mathrm{r}} + n_{\mathrm{r}}n_{\chi}\sigma_1 + \Gamma n_{\chi}, \tag{18}$$

where Γ is the decay width for the χ particles. In the equilibrium

$$mn_{\chi} - \Gamma n_{\chi} - n_{\mathrm{r}}n_{\chi}\sigma - 3Hn_{\chi} = 0, \tag{19}$$

$$-3Hn_{\mathrm{r}} + n_{\mathrm{r}}n_{\chi}\sigma_1 + \Gamma n_{\chi} = 0. \tag{20}$$

If $m \gg \Gamma$, which means decay of the χ particles into light species, this corresponds to the applicability of perturbation theory for calculations of the processes of decay, and as a rule this is valid in models with coupling less than unity. We get (taking into account (13))

$$\rho_{\mathrm{r}} = \frac{m^2}{\sigma}; \quad \rho_{\chi} = \frac{3mH}{\sigma}. \tag{21}$$

The equilibrium density of the relativistic particles ρ_{r} is achieved when the density of evaporated and decayed χ particles,

$$\rho_{\chi\mathrm{d}} \sim \Gamma\frac{1}{m}\rho_{\mathrm{vac}} = \Gamma\frac{1}{m}\frac{m^2\phi_0^2}{8},$$

satisfies the condition

$$\Gamma\frac{1}{m}\frac{m^2\phi_0^2}{8} > \frac{m^2}{\sigma}.$$

This gives the lower limit on the characteristic width of the χ particles decay:

$$\Gamma > \frac{8m}{\phi_0^2\sigma}. \tag{22}$$

If this condition is satisfied, the potential evolves not to the value $V(\delta) = 0$, but to the value $V_{\max} - \rho_{\chi} - \rho_{\mathrm{r}}$, becoming successively flatter. The slow evaporation of the χ condensate acts in such a way as to flatten the potential near its symmetric state.

We see that the back reaction of the evaporated particles and products of their decay produces an effective damping of the scalar field oscillations, which leads to effective flattening of an initially non-flat potential and provides the mechanism responsible for inflation without special fine-tuning of the potential parameters. This mechanism is qualitatively similar to the effective flattening found around the spinoidal line in the Hartree-truncated theory of spinoidal inflation [7] (which involves fine-tuning at the slow-rolling stage preceding the spinoidal regime).

The process of Λ decay is governed by the equation

$$\frac{\mathrm{d}\rho_{\mathrm{vac}}}{\mathrm{d}t} = -3H\left(\rho_{\chi} + \rho_{\mathrm{r}} + \frac{1}{3}\rho_{\mathrm{r}}\right) = -4H\frac{m^2}{\sigma}. \tag{23}$$

Now let us estimate the characteristic time of decay for the two limiting cases of minimal and maximal cross-section σ. The minimal cross-section is $\sigma = 4\pi/m^2$, which is the cross-section for scattering of particles of masses $m/2$ in the hard ball approximation. In this case

$$\frac{\mathrm{d}\rho_{\mathrm{vac}}}{\mathrm{d}t} = -\frac{H}{\pi}m^4. \tag{24}$$

The law for Λ decay is given by

$$\rho_{\mathrm{vac}} = \rho_0\left(1 - \frac{t}{\tau}\right)^2;$$

$$\tau = \sqrt{\frac{3\pi}{2}}\frac{m_{\mathrm{Pl}}\sqrt{\rho_0}}{m^4} = \sqrt{\frac{3\pi}{32\lambda}}\frac{m_{\mathrm{Pl}}}{m^2}. \tag{25}$$

In this case the e-folding number

$$H\tau = \frac{\pi}{8}\frac{1}{\lambda} \tag{26}$$

gives sufficient inflation for reasonable values of the coupling λ. The characteristic time for reheating is λH^{-1} and the reheat temperature $T_{\mathrm{RH}} \sim \lambda^{1/4}m$. The upper bound for σ is determined by $\sigma = \pi/H^2$. In this case

$$\frac{\mathrm{d}\rho_{\mathrm{vac}}}{\mathrm{d}t} = -\frac{4}{\pi}H^3m^2; \quad \rho_{\mathrm{vac}} = \frac{\rho_0}{(1+t/\tau)^2}, \tag{27}$$

where

$$\tau = \left(\frac{3^3}{2^{11}\pi}\right)^{1/2}\frac{1}{\lambda^{3/2}}\left(\frac{m_{\mathrm{Pl}}}{\phi_0}\right)^4\tau_{\mathrm{Pl}}. \tag{28}$$

The e-folding number is then

$$H\tau = \frac{3}{32}\frac{1}{\lambda}\left(\frac{m_{\mathrm{Pl}}}{\phi_0}\right)^2, \tag{29}$$

and, for the considered case $\phi_0 \ll m_{\mathrm{Pl}}$, inflation is sufficient for any λ. The reheating temperature is $T_{\mathrm{RH}} \sim \lambda^{1/4}H$.

A more detailed investigation of the dynamics of vacuum decay needs a particular model for calculating σ, but the results will be within the range between the cases of minimal and maximal σ_1. For example, the picture of evaporation investigated in [12] corresponds to the case of evaporating of Higgs bosons and their reheating to the Hawking temperature $T_{\mathrm{RH}} \sim H$. The rate for Λ decay is given in this case by

$$\frac{\mathrm{d}\rho_{\mathrm{vac}}}{\mathrm{d}t} = -3Hm^{5/2}H^{3/2}. \tag{30}$$

Here we considered Λ decay in the case of vacuum dominance. When the radiation density starts to exceed the vacuum density at the last stage of evaporation, we would have to change (16) taking into account the evolution of the Hubble parameter as well as of matter and the radiation density, which in the standard FRW cosmology evolves as t^{-2}. Equation (20) reproduces this behavior, starting at the stage of vacuum dominancy for the case of the maximum possible cross-section σ. Provided that this behavior remains dominating at successive stages, this corresponds to the existence of a remnant evaporating condensate today with a density comparable to the total density in the universe, which seems to agree with the results of a recent analysis of the observational data [1].

The generalization of this approach to the case of an arbitrary scalar field potential is straightforward. Any cosmologically reasonable potential must satisfy the condition $V(\phi^2 - \phi_0^2) > 0$. The true vacuum state $\langle\phi\rangle = \phi_0$ is determined as the minimum of the potential $V = 0$. The physical particles $\chi = \phi - \phi_0$ are defined over the true vacuum state. Their mass is given, as usual, by $\partial^2 V/\partial\phi^2$. We treat any state with $V(\chi) > 0$ as a bound state of χ particles trapped inside a Bose condensate. The equilibrium fluctuation density corresponds to a deviation of the potential from its initial value at the given point by the quantity $\rho_\chi + \rho_{\mathrm{r}}$ (see (14)). This means that in a characteristic time m^{-1} the field has not completely moved to its ground state, but, instead, is stabilized near its initial value, having slightly changed by the magnitude $\rho_\phi + \rho_{\mathrm{r}} \ll V(\chi)$.

We see that the decoherence of the χ particles and the back reaction of their decay products leads to effective freezing of the field near its initial value. Near this value the potential becomes locally flat, and the energy density of the condensate of χ particles starts to play the role of an effective cosmological constant. It realizes the case of chaotic inflation for an initially non-flat potential in the case $m \gg H$ and $\phi_0 \ll m_{\mathrm{Pl}}$.

At first sight, the appearance of a slow evaporation regime in our approach seems to lead to the same spectrum of initial density fluctuations as in slow-rolling models. However, the origin of the fluctuations is different. In our case, fluctuations are generated by a statistical distribution of evaporated particles, while in the typical slow-rolling picture they originate from non-simultaneous transitions to the ground state. We are currently investigating this problem and we expect a "blue" spectrum of density fluctuations, which seems to be favored by theories of large scale structure formation.

5 Discussion

The approach proposed here is based on the interpretation of the cosmological term Λ as the vacuum energy density of physical scalar particles in their ground state, which is a bound collective state of the Bose–Einstein condensate. Such an interpretation has a transparent physical meaning in the context of the Higgs mechanism of spontaneous symmetry breaking. In this case, the vacuum energy density of the Higgs field in a symmetric state, $V(0)$, is related to the self-interaction energy of massive scalar particles bound within a condensate.

Formally, the mechanism presented in this paper is based on the solution describing cosmological evolution in terms of Higgs field dynamics, which was somehow overlooked in the literature. Being rather trivial from the mathematical point of view, this solution appears to make non-trivial physical sense as corresponding to the kinetic equilibrium regime for the slow evaporation of a Bose–Einstein condensate. This kind of solution has analogies in experimentally studied Bose–Einstein condensation in atomic physics [35].

Our solution describes the effective damping of scalar field oscillations, which leads to effective flattening of the initially non-flat scalar field's effective potential driving inflation. This flattening results naturally from the mechanism involving kinetic equilibrium in the process of Λ decay as Bose condensate evaporation, and thus no ad hoc fine-tuning is needed.

Let us note that the considered Higgs case can be treated as simple illustration of the idea, which seems to be more generic and can find applications also in the case of non-Abelian gauge models without Higgs mechanism, in which symmetry breaking is induced by non-linearity of the gauge interactions similar to confinement in QCD.

Dependent on the relation between masses and values of the scalar field potentials in the symmetric state, different non-trivial realizations of the scenario are possible. If the value of the potentials in the symmetric phases (effective Λ driving inflation) of the first and the second breakings are close, while the masses differ substantially, then the first breaking can terminate in the de Sitter stage dominated by the value of effective Λ for the second breaking. If masses for the second and the next breakings satisfy the condition $m > H$, then the process of evaporation triggers simultaneous breaking of the second and next symmetries.

We can conclude that the mechanism of slow evaporation provides a physically self-consistent inflationary model based on incorporating the origin of the cosmological constant in the dynamics of spontaneous symmetry breaking and is able to incorporate reheating and a proper primordial perturbation spectrum as well as a non-zero cosmological constant today. Currently we are working on the rigorous justification of the existence of this mechanism in the framework of quantum field theory [15].

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